Stability Effect for Passive Dynamic Walker by Stance Leg Heel Angle Constraint Using Foot Shape

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Abstract: One of the methods known to stabilize a passive dynamic walk is to restrict a hip joint angle. The hip restriction method effects as resetting the walking dynamics every time at the beginning of the walk. A Constraining Foot Shape is known to give the same effect by a more compact configuration. However, it was not clear how much this dynamics resetting effect will be given for a specific design parameter such as a length of the foot. This paper shows a numerical approach for this. Firstly, we show a dynamics of walking by 2D stick model with foot length. This is realized by considering a support point exchange effect. Secondly, we show a simulation results by this model. From these, we show that a longer foot will give a more strong effect of resetting walking dynamics. Finally, we demonstrate the validity of the model by making an actual hardware of 2D stick model with foot length. In this model, a scuffing effect was avoided by using landing islands on a slope.

Keywords: Passive Walk, Constraining Sole Shape, Foot length, Stability

1. INTRODUCTION

Passive dynamic walk is one of the movement schemes for a biped walking robot, which do not use actuators to drive its leg movement and walks down a slope only by using the gravitational force [1, 2]. However, a passive dynamic walk is highly sensitive to its parameters. A small fluctuation of parameters such as slope angles and initial swing velocities will cause a failure to continue walking.

A stable walk is regarded as a dynamical system that has the same state variables between adjacent walk phases. In this context, an efficient way to a stabilized walk is to reset the walking mechanism’s state variables to its initial values at every beginning of a walk phase [5]. To this end, Sano et al. introduced a mechanical limiter that restricts hip joints (Fig. 1) [5]. This simple mechanism keeps the same posture of legs every time at the beginning of a stance phase, which is known to attract its dynamics to a stable limit cycle.

However, a hip joint angle limiter does not restrict the angle between a stance leg and a floor. This causes the robot to turn over as shown in Fig. 2. This figure shows the case where the velocity of a robot’s swing leg is too fast. In this case, the hip joint reaches the restricted angle, while the entire body leans over and will fall down.

To give the same effect of restricting hip joint angle while avoiding the above situation, we proposed a simple idea of restricting a heel angle by a sole shape [6].

Fig.3 shows a concept of the restriction by foot. Suppose a sufficiently long foot which is rigidly attached at the stance leg with a preset angle. If the foot is long enough, the heel of the stance leg will no more rotate beyond the preset angle. This also limits the angle between the stance leg and the line perpendicular to the slope, which is part of the hip joint angle.

After a few moment, the swing leg will touch on the ground (Fig.3 right). Since the heel angle of the stance leg is fixed, the heel angle of the swing leg landing at the floor will also be the same at every time. This restricts the other half of the hip joint as in Fig.3. Therefore, the swing leg heel angle restriction by a long foot will give the same effect as the hip joint restriction

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However, in real situations, we cannot expect sufficiently long foot that will be able to stop the heel rotation. Instead, we must consider a short foot length. In this case, the robot will keep turning at its toe as in Fig. 4.

This process is summarized in the following three phases:

1. A stance leg rotates at its heel (Fig. 4 left).
2. A foot at the stance leg touches on the ground (Fig. 4 middle).
3. A stance leg rotates at its toe (Fig. 4 right).

A big difference between (2) and (3) is the change of the center of rotation. The center of rotation moves away from its original position toward the robot’s moving direction. Just before changing the center of rotation, the toe will produce a repulsive force that bothers the rotation of the stance leg. This infers that the change of the center of rotation will absorb the angular momentum of the entire robot.

The change of the center of rotation only occurs when a swing leg continues moving after a stance leg reaches its heel restriction angle. This means that the robot will converge to a stable walk where the leg change will occur at the heel restriction angle.

To examine these assumptions, we made a simple 2D dynamics model for a knee-less passive biped that has toes. By numerical simulations, we were able to show that the biped with foot and toe actually effects to pull its angular velocity and leg angles back to their initial values. Also we demonstrated its effect by making a hardware of the 2D model with toes. From observation of the experiments, it was shown that the existence of toes surely affects to stabilize walking. In this model, a scaffolding effect was avoided by using landing islands on a slope.

2. A SIMPLE DYNAMICS MODEL OF A BIPED WITH FEET AND TOES

2.1 A model and its parameters

We consider a 2D biped without knee and with feet as in Fig. 5.

The parameters are as follows:

- $g$: Gravitational force
- $L_1$: Length of stance leg
- $L_2$: Length of swing leg
- $\theta_1$: Angle between the stance leg and a line perpendicular to the slope
- $\theta_2$: Angle between the swing leg and a line perpendicular to the slope
- $m_1$: Mass at hip point
- $m_2$: Mass at the heel of the swing leg
- $\alpha$: Angle between a leg and a foot
- $\gamma$: Slope inclination
- FLStance: Length of foot at stance leg
- FLSwing: Length of foot at swing leg

Note that the mass of links are assumed to be concentrated at the hip and the heels of legs. Also the foot is fixed on the leg by the angle $\alpha$.

2.2 Equations of motion

The model is regarded as a double pendulum. Therefore, the equations of motion are simply written as follows:

$$\dot{\theta}_1 = \theta_1$$
$$\dot{\theta}_2 = \theta_2$$

$$\dot{\theta}_1 = \frac{-m_1 L_1^2 \gamma \sin \theta_2 + m_2 L_2 \gamma \theta_2^2 \sin \theta_1 + (m_1 + m_2) g L_1 S_1 + m_2 g L_2 S_2}{(m_1 + m_2) L_1^2 - m_2 L_2^2 \gamma^2}$$

$$\dot{\theta}_2 = \frac{-(m_1 + m_2) L_1 \sin \theta_2 + m_2 L_2 \gamma \theta_2^2 + (m_1 + m_2) g S_2 + (m_1 + m_3) g S_3}{-(m_1 + m_2) L_2^2 + m_2 L_2^2 \gamma^2}$$
where $C$, $S$, $S_2$, $S_3$ correspond to the followings for simplicity:

$$C = \cos(\theta_1 + \theta_2)$$
$$S = \sin(\theta_1 + \theta_2)$$
$$S_2 = \sin(\theta_1 + \gamma)$$
$$S_3 = \sin(\theta_3 - \gamma)$$

### 2.3 Explanation of the equations

These equations describe the behavior of the model from the beginning of a swing phase. At the moment when a toe touches on the ground (Fig.4 middle), the change of the center of rotation occurs. This is described as follows (Fig.6):

1. The length of the stance leg changes as $L_1 \rightarrow L_1^+$. (1)
2. The angle and the angular velocity of the stance leg at the hip point change as $\theta_1 \rightarrow \theta_1^+$ and $\dot{\theta}_1 \rightarrow \dot{\theta}_1^+$. (2)
3. The length of the stance leg changes as $h \rightarrow h^+$. (3)
4. An angular velocity for a new stance leg hip angle is defined. We describe this as $\theta_1^+$. (4)

The calculation of these values are as follows:

1. The stance leg hip angle $\theta_1$ at the time when the toe touches on the ground is derived by:

   $$\alpha = 2\theta_1 - \frac{\pi}{2}$$

   Note that $\alpha$ is preset as a design factor.
2. The new length for the stance leg $L_1$ is derived as a length between the hip joint and the toe $(L_1^+)$. (2)
3. By this new stance leg, a new stance leg hip angle is defined. We describe this as $\theta_1^+$. (3)
4. An angular velocity for a new stance leg hip angle should be calculated. This is done by a law of conservation of angular momentum. The result is:

   $$\dot{\theta}_1^+ = \frac{L_1^2 + L_1^+ L_1^+}{L_1^+ + L_1^-} \dot{\theta}_1$$

After the center of rotation has changed at the toe, the equations of motion is used with these new state values. When the swing leg touches on the ground, the state values should be changed according to a well-known biped simulation [7, 8].

### 3. SIMULATION RESULTS

A series of numerical experiments were conducted by using Scilab software. A snapshot of a typical walking animation and the plot for hip joint angle are shown in Fig.7 and 8. The simulation parameters are as follows:

- $g$: 9.80665 (m/s²)
- $L_1$: 50 (cm)
- $L_2$: 50 (cm)
- $m_1$: 3 (kg)
- $m_2$: 0.1 (kg)

Fig.8 is plotted under the conditions of the foot length 50 [cm] and the slope inclination 0.009 [rad]. In this figure, the angle rapidly drops around 13 [s] and 14 [s]. At this moment, the change of the center of rotation was done. After the change, a new (virtual) stance is from the toe to the hip joint. Therefore, the value of the angle jumped to a newly defined value. The continuity of the actual angle is kept at this changing point.

An important property in this result is that, after the changing point, the value of the angle first decreases a little bit and then increases again. This is explained as follows: (1) After the toe touches on the ground, the stance leg keeps its rotation but the velocity of rotation keeps decreasing. (2) Then, the stance leg stops at a certain angle. (3) After that, the stance leg begins to rotate back to the angle at which a foot (toe) touches on the ground. By this, it should be noted that the existence of foot and toe effects a certain amount for bringing the robot's dynamics back to the same point.

Next, to observe the relationship between the angle of slope and the length of foot, we conducted some numerical experiments and calculated an error rate $E$ as follows:

$$E = (\theta_1 - \theta_1^+)^2 + (\dot{\theta}_1 - \dot{\theta}_1^+)^2$$
This value indicates a distance of state variables between two adjacent walking phases. It is used as a measurement of how close the state is after one walking phase.

Fig. 9 shows the result. The graph was plotted by three different sizes of foot (10[cm], 30[cm], and 50[cm]). The inclination of slope was changed from 0.009[rad] to 0.036[rad]. The change of inclination is regarded as an environmental disturbances. From this plot, it should be concluded that the longer foot size contributes to reduce the error rate.

![Fig.9. Plot of error rate at different foot size.](image)

To clarify this result, we calculated the gradient of error rates against foot sizes by using data in Fig.9. Fig.10 shows the graph. From this figure, the effect of error reduction by longer foot size was shown.

![Fig.10. Plot of gradient of error rate against foot size.](image)

**4. A HARDWARE EXPERIMENTS FOR 2D BIPED WITH FOOT AND TOE**

We examined the effect of the foot and toe from numerical simulations as described above. However, it is needed to get a proof of the correctness of the model. To this end, we made an actual hardware that is close to the proposed model, and conducted a series of experiments under real environments. Fig.11 shows an experimental hardware.

The hardware consists of a pair of outer legs and another pair of inner legs. Some weighting materials are attached to keep the same inertia with two sets of legs.

One of the big problems of the real-world experiments by this stick model is that the top of swing legs will scuff the slope. To avoid this, we placed ‘landing islands’ on a slope (Fig.12). A waking machine will swing its legs below the level of the landing islands. Since the islands are having some heights above the slope, the swing movement is not bothered.

![Fig.12. A walking slope realized by ‘landing islands.’](image)

Fig.13 shows an actual slope (3[deg]) with landing islands. Note that the distance between landing islands (blue block) depends on an actual stride by the hardware.

![Fig.13. An actual walking slope with landing islands.](image)

By this environment, we conducted experiments with the hardware with foot (toe) and without foot (toe). The hip joint angle and the time of one stride were measured.
Fig. 14 shows snapshots of the actual experiments.

![Figure 14](image_url)

Fig. 14. Experiments by the 2D model with (right) or without (left) foot length.

White lines in Fig.14 (below) shows the hip joint angles for the hardware with and without foot and toe. It is observed that the hip angles are mostly the same with two types.

Instead, the duration of one stride by the hardware with or without foot largely differed with each types. These were 5.44[s] and 4.03[s] respectively. This means that the foot and toe contributed to extend the cycle time of strides. It seems the foot and toe worked as braking the exceeding swing speed in this case.

5. CONCLUSION

This paper described the effect of foot and toe for the stabilization of passive dynamic walk. We proposed a simple stick model with toes and derived some results showing that a rigidly attached foot has an effect of bringing the walking dynamics back to a same state.

Although the efficiency was given, we have not yet shown how to find a proper heel restriction angle according to a given walking environment. This is a hard problem and may only be given by simulations of the targeting robot.

The heel angle restriction begins to work at a condition where a stance leg inclines at a certain angle. Therefore, it should be pointed out that a foot attached with a sufficiently small heel angle will work to give a braking effect in cases where the robot moves too fast. Also it does not bother the walking unless a stance leg reaches this critical heel angle. By these, a heel restriction angle may not be so sensitive and will be given by a designer’s instinct. Of course this should further be discussed.

There are also lacking extensive numerical experiments for many cases of simulation parameters. We were currently working hard to further clarify our results.

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